Probabilistic Foundations of Artificial Intelligence

Problem Set 1

Oct 3, 2014

1. Propositional Logic and Resolution

A propositional 2-CNF expression is a conjunction of clauses, each containing *exactly* 2 literals, e.g.,

$$(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G).$$

(a) Prove using resolution that the above sentence entails G.

(b) Two sentences are *semantically distinct* if they are not logically equivalent. How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?

(c) Using your answer to (b), prove that propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.

(d) Explain why your argument in (c) does not apply to 3-CNF.

2. First Order Logic

(i) Write down a satisfiable logical sentence in First Order Logic, such that every world in which it is true, contains exactly one object.

(ii) Now, consider a vocabulary that contains the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
$Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in First Order Logic:

(a) Emily is either a surgeon or a lawyer

(b) Joe is an actor, but he also holds another job.

(c) All surgeons are doctors.

(d) Joe does not have a lawyer (i.e. Jose is not a customer of any lawyer)

(e) Emily has a boss who is a lawyer.

(f) There exists a lawyer whose customers are all doctors.

(g) Every surgeon has a lawyer

3. FOL Field Guide

Suppose you wish to create an "intelligent field guide" that helps people identify bird species. To reason about how different attributes relate to the bird species, you decide to employ First Order Logic. Consider the following useful facts.

Birds are winged animals, which are characterized by feathers and beaks. Bob is a bird watcher, but he doesn't like birds with long beaks. All woodpeckers have long beaks. One day, Bob finds a red-headed bird. He likes the bird very much, and names it as Aly. A bird with red head is either a pileated woodpecker or a cardinal.

(i) From the set of facts listed above, extract useful sentences that could be used to infer the correct bird species of Aly. Write down the sentences in English.

(ii) Transform the English sentences into a First Order Logic knowledge base.

(iii) Is Aly a Cardinal? Answer the question using propositionalization, followed by resolution.