

Probabilistic Foundations of Artificial Intelligence

Problem Set 2

Oct 10, 2014

1. Finding the fake coin

Suppose you are given a bag containing n unbiased coins. You are also told that $n - 1$ of these coins are normal, that is, they have a head on one side and a tail in the other. The remaining one is fake and has heads on both sides.

- (a) Suppose you pick a coin from the bag uniformly at random, you flip it, and get a head. Given this result, what is the probability that the coin you picked is the fake one? (Note that we ask for a conditional probability.)
- (b) Suppose you continue flipping the same coin for a total of k times and you get k heads. What is the probability that you picked the fake coin?
- (c) Now, suppose you devise the following method to determine if the coin is fake or not. You flip it k times, after which you conclude that it is the fake one if all k flips have resulted in heads, else you conclude that it is normal. What is the probability that using this method you arrive at a wrong conclusion? (Note that this time we ask for an unconditional probability.)

2. Naive Bayes

Suppose you have a bag of three biased coins a , b , and c , with probabilities of coming up heads of 0.2, 0.6, and 0.8 respectively. You draw a coin uniformly at random from the bag and flip it three times to generate the sequence of outcomes X_1, X_2, X_3 .

- (a) Draw the Bayesian Network corresponding to this setup and specify the necessary Conditional Probability Tables (CPTs).
- (b) Calculate which coin was most likely to have been drawn from the bag, if two of the observed outcomes were heads and the other was a tail.