Probabilistic Artificial Intelligence

Problem Set 1

Sep 29, 2017

1. Conditional Probabilities

For each statement below, either prove it is true, or give a counterexample showing it is false.

   (a) If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$

   (b) If $P(a|b, c) = P(a)$, then $P(b|c) = P(b)$

   (c) If $P(a|b) = P(a)$, then $P(a|b, c) = P(a|c)$

2. Finding the fake coin

Suppose you are given a bag containing n unbiased coins. You are also told that $n - 1$ of these coins are normal, that is, they have a head on one side and a tail in the other. The remaining one is fake and has heads on both of its sides.

   (a) Suppose you pick one coin from the bag at random and then you flip it and get a head. Given this result, what is the probability that the coin you choose is the fake coin? (Note we are asking for a conditional probability).

   (b) Suppose you continue flipping the same coin for a total of k times and you see k heads. Now what is the conditional probability that you picked the fake coin?

   (c) Now suppose you devise the following method to determine if the coin is fake or not. You flip it k times, and your method returns it is a fake coin if all k flips result in heads. Otherwise the method returns that it is a normal coin. What is the probability that this method returns an error? (note this time we ask for an unconditional probability)

3. Geometric Distribution

Suppose you throw a dice repeatedly until you get a 6.

   (a) What is the probability of finding a sequence of length n?

   (b) What is the expected value of the sequence length?

   (c) What is the expected number of 3s we observe?