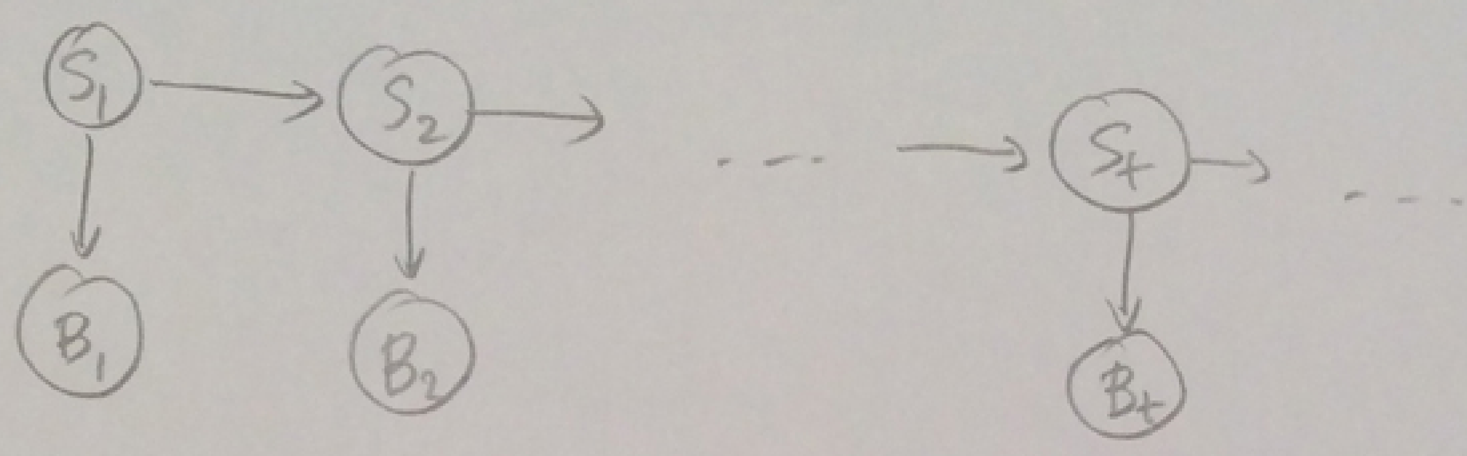
Problem 3, Exam 2016

(i)



S_t : Sun or not on day t (T, F).

B_t : Bob goes to the beech (T) or to the university (F) on day t.

$\boxed{S_1}$

S_1	T	F
	0,95	0,05

$\boxed{S_{t+1} \mid S_t}$

$S_t \backslash S_{t+1}$	T	F
T	0,9	0,1
F	0,6	0,4

$\boxed{B_t \mid S_t}$

$S_t \backslash B_t$	T	F
T	0,8	0,2
F	0,1	0,9

(ii) Single sample is two vectors in $\{T, F\}^{30}$.

Pseudocode:

$s_1 \sim P(S_1)$
$b_1 \sim P(B_1 \mid S_1 = s_1)$
for $t = 2, \dots, 30$:
  $s_t \sim P(S_t \mid S_{t-1} = s_{t-1})$
  $b_t \sim P(B_t \mid S_t = s_t)$
return $((s_1, \dots, s_{30}), (b_1, \dots, b_{30}))$

(iii) For each sample X_j $(j = 1, \dots, N)$ from the HMM, extract the "beach" part $(b_1^{(j)}, \dots, b_{30}^{(j)})$ and count the number of days Bob went to the university:

$$\boxed{u^{(j)} := \sum_{i=1}^{30} \mathbb{1}_{\{b_i^{(j)} = F\}}}$$

To get an estimete over all samples, compute average

$$\boxed{u := \frac{1}{N} \sum_{j=1}^{N} u^{(j)}}$$