Probabilistic Artificial Intelligence

Problem Set 1

September 28, 2018

1. Conditional Independence

Consider the following joint distribution for three random variables, $a, b, c \in \{0, 1\}$.

a	b	c	$p(a, b, c)$
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

Show that a and b are dependent, namely $p(a, b) \neq p(a)p(b)$. But, they are marginally independent given c, namely $p(a, b \mid c) = p(a \mid c)p(b \mid c)$. (c.f. Bishop *Pattern Recognition and Machine Learning*, Exercise 8.3)

2. Bayes Rule

A routine breast cancer mammography screening is performed on a group of people of age fourty. 1% of the participants in the screening actually have breast cancer. 80% of the people in the screening with breast cancer received positive results (has breast cancer) on the mammmography test. 9.6% of people without breast cancer received a positive result on their mammographies. Suppose a person of this age receives a positive result on their mammography. Given the information in this screening, what is the probability that he has breast cancer?

3. Chain rule

Derive the chain rule from the basic rules of probability. (Hint: by the definition of conditional probability $P(A, B) = P(A \mid B)P(B)$). How many factorizations are possible for a distribution on n random variables?