Probabilistic Artificial Intelligence

Problem Set 4

Nov 9, 2018

1. Bayesian networks and Markov chains

Consider the query $P(R|S = t, W = t)$ in the following Bayesian network, and how Gibbs

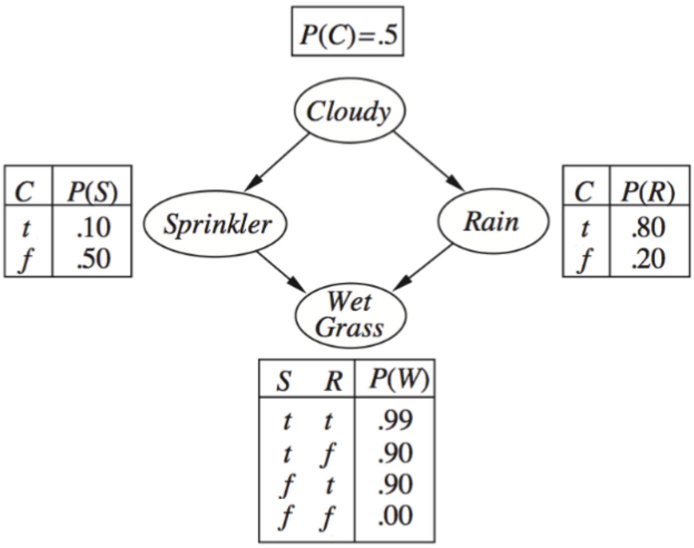

Figure 1: Bayesian Network

sampling can answer it.

(i) How many states does the Markov chain have?

(ii) Calculate the transition matrix T containing $P(X_{t+1} = y \mid X_t = x)$ for all x, y.

(iii) What does T^2, the square of the transition matrix, represent?

(iv) What about T^n as $n \to \infty$?

(v) Explain how to do probabilistic inference in Bayesian networks, assuming that T^n is available. Is this a practical way to do inference?

2. Markov chains and detailed balance

Assume that you are given a Markov chain with state space Ω and transition matrix T, which is defined for all $x, y \in \Omega$ and $t \geq 0$ as $T(x, y) := P(X_{t+1} = y \mid X_t = x)$. Furthermore, let π be the stationary distribution of the chain.

(i) Show that, if for some t the current state X_t is distributed according to the stationary distribution and additionally the chain satisfies the detailed balance equations

$$\pi(x)T(x, y) = \pi(y)T(y, x), \text{ for all } x, y \in \Omega,$$

then the following holds for all $k \geq 0$ and $x_0, \ldots, x_k \in \Omega$:

$$P(X_t = x_0, \ldots, X_{t+k} = x_k) = P(X_t = x_k, \ldots, X_{t+k} = x_0).$$

(This is why a chain that satisfies detailed balance is called *reversible.*)

(ii) Show that, if T is a symmetric matrix, then the chain satisfies detailed balance, and the uniform distribution on Ω is stationary for that chain.