Efficient Minimization of Decomposable Submodular Functions

By Peter Stobbe (speaking) and Andreas Krause

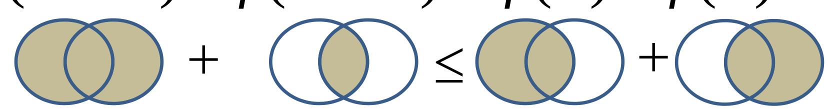
KEY PROPERTIES FOR MINIMIZATION

Continuous optimization: **convexity**

Discrete optimization: **submodularity**

Real-valued functions of sets
eg. Classifying pixels,
clustering data points,
many more

$$f(A \cup B) + f(A \cap B) \le f(A) + f(B)$$



General submodular minimization : $O^*(n^5)$

Some restrictive classes (pairwise interactions): efficient

NEW CLASS OF SUBMODULAR FUNCTIONS!

DECOMPOSABLE

$$f(A) = \sum_j \phi_j\left(\sum_{k \in A} w_j[k]\right)$$

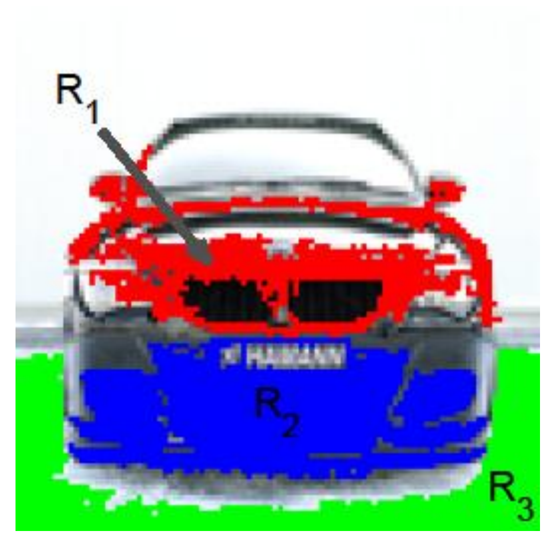
ϕ_j concave

$\mathbf{w}_j \geq \mathbf{0}$

Need a decomposition of the function into a sum of simpler functions

Example: Sum of higher order potentials for MAP inference of Markov Random Field

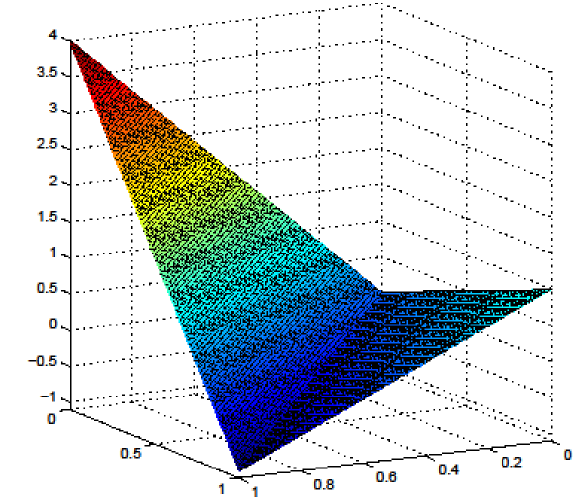
$$f(A) = \sum_j \left|R_j \setminus A\right|\left|R_j \cap A\right|$$



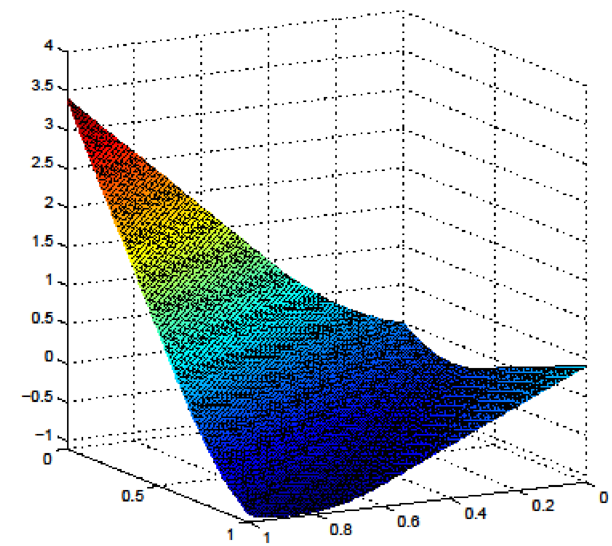
OVERVIEW OF

Smoothed Lovàsz Gradient (SLG)

1. Formulate as a convex optimization problem *(Lovàsz 1980)*



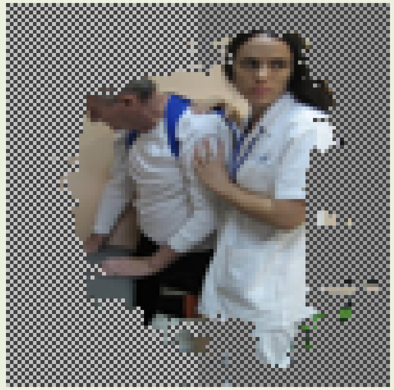
2. Apply modern techniques for general nonsmooth convex optimization *(Nesterov 2004)*
   Decomposition makes it possible!



3. Use a novel stopping criterion to finish early with optimal answer for discrete problem

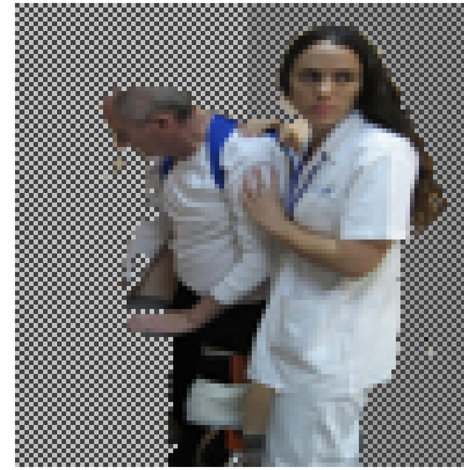
LET'S SEE SOME RESULTS!

Original

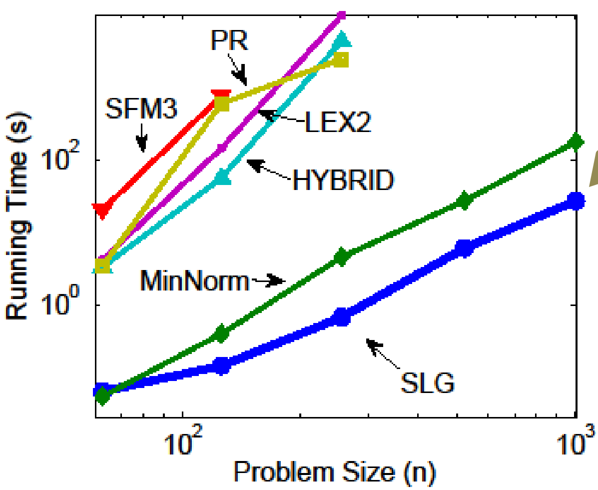


No potentials

Pairwise (Ising) potentials



Higher order potentials
(Made possible by SLG)



Matches or outperforms other general-purpose algorithms on standard test problems

CAN SOLVE PROBLEMS WITH 10,000 VARIABLES IN A MINUTE.